

How does the radius of an electrical coil affect the electromotive force (EMF) induced in it when a magnet is passed through the coil at a constant velocity?

Word Count: 3000

Introduction

My interest in building electronics projects involving microcontrollers, sensors, motors, and printed circuit boards led me to explore electromagnetic induction and its applications across technologies including generators, sensors, and wireless chargers. Curious about understanding how the properties of a coil affect induced electromotive force (EMF), I conducted preliminary tests with magnets and coils I had available at home, noticing that larger diameter coils seemed to produce a larger magnitude of EMF.

By researching Faraday's Law, Ohm's Law, and the formula for magnetic flux linkage, I found an opportunity to confirm a direct relationship between the coil radius squared and the induced EMF.

In this experiment, a magnet moves vertically upwards through a solenoid at a constant speed using a linear actuator, which ensures a constant rate of change of magnetic flux linkage and minimizes random errors that could arise from manual movement of the magnet or dropping the magnet in free fall.

I chose to use a coil of wire rather than a solid metal pipe because, in a coil, induced current flows in a well-defined circuit that makes the EMF easier to measure. In a solid metal pipe, the magnetic field would induce circulating currents called eddy currents within the material of the pipe. Eddy currents would form closed loops within the pipe and flow in directions that oppose the change in magnetic flux due to Lenz's Law (Ling, Samuel J., et al 2018), dissipating energy as heat which would reduce the total measured EMF. Also, eddy currents would spread throughout the pipe unpredictably, rather than in a defined circuit, and can create their own magnetic fields that can interfere with that of the magnet. By using a coil of wire instead of a solid pipe, the current is restricted to a specific path, minimizing the impact of eddy currents on the measured EMF.

Theory:

Electromagnetic induction is the phenomenon by which a changing magnetic field can induce an EMF in a conductor (Kinsler, 2020). Faraday's Law states that the induced EMF in a closed circuit is directly proportional to the rate of change of magnetic flux linkage through the circuit (Anderson et al., 2020).

Faraday's Law of Induction:

$$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$$

Where ε = induced EMF (V), N = number of turns in the coil (unitless), $\Delta\Phi$ = change in magnetic flux linkage (Wb), and Δt = change in time (s).

Conceptual definition of Magnetic flux linkage Φ when a magnet passes through a solenoid or a coil, which can be calculated using the following formula (Halliday et al., 2018):

$$\Phi = B \cdot A = B \cdot \pi r^2$$

Where B = magnetic field strength (T), A = cross-sectional area of the coil (m^2), and r = radius of the coil (m)

The negative sign indicates the direction of the induced EMF as described in Lenz's Law, which states that the direction of the induced EMF is such that the induced current must create a magnetic field that opposes the change in magnetic flux linkage (Purcell & Morin, 2013).

In a circular coil, the cross-sectional area A can be calculated using the following equation:

$$A = \pi r^2$$

Substituting above equation for area A :

$$\varepsilon = -N \frac{\Delta(B\pi r^2)}{\Delta t} = -N\pi r^2 \cdot \frac{\Delta B}{\Delta t}$$

This shows that the EMF is directly proportional to the radius of the coil squared. Plotting the magnitude of induced EMF against coil radius squared should yield a linear relationship with a slope equivalent to $N\pi \cdot \frac{\Delta B}{\Delta t}$, which will be constant because N and $\frac{\Delta B}{\Delta t}$ do not change during the experiment.

As the coil radius increases, its cross-sectional area increases proportionally to the square of the radius ($A = \pi r^2$). Conceptually, this larger area allows more magnetic flux to pass through the coil when the magnet moves through it, so for the same rate of change in the magnetic field, the change in magnetic flux linkage $\Delta\Phi$ will be greater, inducing an EMF of greater magnitude according to Faraday's Law.

Use of a Linear Actuator

To ensure a constant speed, the magnet is moved by a linear actuator controlled with a fixed pulse-width modulation (PWM) signal from an Arduino-driven motor controller. This helps avoid error produced by manually moving the magnet through the coil or due to gravitational acceleration if dropping the magnet in free fall. This setup helps ensure that the rate of change of magnetic field strength is constant throughout all trials.

Theoretical Limitations

The above equation assumes a uniform magnetic field at all distances, which is unrealistic because an infinitely large coil radius would therefore generate an infinite EMF. In reality, however, as the coil radius increases, a larger portion of the coil is exposed to regions of weaker magnetic field strength, which would cause the measured EMF to be lower than predicted. Depending on the magnet, within a close range of distances, the magnetic field strength is relatively constant. However, beyond a certain distance the magnetic field strength begins to decrease proportionally to the inverse cube of the distance ($B \propto \frac{1}{d^3}$) ("Magnetic Fields Varying as an Inverse Cube," Millersville University). This experiment assumes that, for the range of coil radii used, the magnetic field is uniformly distributed and the magnetic field strength is approximately constant within the coil's radius.

Research Question:

How does the radius of an electrical coil affect the electromotive force (EMF) induced in it when a magnet is passed through the coil at a constant velocity?

Exploration

Hypothesis:

The induced EMF ε will increase linearly with the square of the coil radius r^2 when a constant rate of magnetic field change is applied when a magnet passes through coils of different radii.

The equation derived above indicates a directly proportional relationship between the magnitude of induced EMF and the coil radius squared. Thus, plotting induced EMF (ϵ) against coil radius squared (r^2) should yield a linear graph with a positive slope that passes through the origin.

The slope of this graph, equivalent to the expression $N\pi \cdot \frac{\Delta B}{\Delta t}$, can be used to verify this relationship. All the coils have 20 turns, so N is constant. Additionally, the rate of change of the magnetic field strength $\frac{\Delta B}{\Delta t}$ will be constant in the experiment because the magnet will be traveling at a constant velocity and the part of the magnetic field that interacts with the coil is uniform.

To calculate the theoretically correct value for the slope, I determined $\frac{\Delta B}{\Delta t}$ to be approximately $3.2 \cdot 10^{-4} \pm 10^{-4} T/s$ by passing the magnet past a hall sensor with the linear actuator at a known speed (0.5 cm/s). I used it to calculate the theoretical slope to be approximately $0.02 \pm 0.01 mV/mm^2$.

Variables:

Variables	What Was the Variable	How Was It Measured
Independent Variable	Coil radius r (cm)	Diameter measured with electronic caliper, then divided by two to calculate the radius. Five coils with radii: 1.270cm, 1.397cm, 1.524cm, 1.651cm, and 1.778cm. Increments chosen based on available manufacturing capabilities to minimize cost.
Dependent Variable	Induced EMF ϵ (V)	Measured with a multimeter set to DC potential difference, with one probe at either end of the coil. Maximum EMF was recorded.

Constants:

Constants	Description	Potential Impact if Variable was Uncontrolled
Magnetic Field Strength in the coils (B), measured in T	The magnetic field strength is kept constant by only measuring EMF when the magnet's center (where the field is most uniform) is aligned with the coil.	Variation in magnetic field strength would introduce random error in the data and affect the magnetic flux linkage, making it harder to isolate changes in EMF due to solely coil radius.
Rate of Change of Magnetic Field Strength ($\frac{\Delta B}{\Delta t}$), measured in T/s	A linear actuator moves the magnet at a constant actuator, ensuring the rate of change of magnetic field strength is constant.	A varying rate of change of magnetic field will cause random error and inconsistent EMF measurements.
Number of turns in Coil (N)	All coils are verified to have the same number of turns	Different number of turns in the coil would change the induced EMF, it is directly proportional to the number of turns.

<p>Coil Material</p>	<p>All coils are made from the same copper alloy and have the same coating.</p>	<p>Variation in material would lead to variation in resistivity and thus resistance in the coils, varying the induced EMF. With higher resistances, the induced EMF will be higher. With lower resistances, the induced EMF will be lower.</p>
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Materials:

- Copper coils: 5 coils made from 0.2588cm ± 0.0001cm thick wire (standard size) with diameters 22.89 mm, 31.75 mm, 49.53 mm, 53.34 mm, 59.69 mm
- 1 Caliper to verify coil diameter (uncertainty ± 0.01 mm)
- 1 Multimeter capable of measuring DC potential difference in mV (±0.1 mV)
- 2 wires with alligator clips to connect multimeter to coil
- 1 Neodymium magnet to pass through the coil
- 1 Linear actuator to move the magnet at constant speed
- 1 Arduino microcontroller to control the linear actuator via PWM
- 1 L298N motor controller to drive the linear actuator
- 1 ring stand to hold the coil in place
- Vernier *LoggerPro* software for graphing the relationship between induced EMF and coil radius
- Duct tape to held secure the magnet
- Cell phone camera to record multimeter readings
- 1 3D printed adapter to hold the magnet in place

Procedure:

1. Verify magnetic field
 - a. Place magnet on paper, mark distances 2 cm, 4 cm, and 6 cm from its surface
 - b. Use Hall sensor to measure the magnetic field at the 2 cm, 4 cm, and 6 cm markings
 - c. Record each measurement to ensure the magnetic field strength is approximately the same.
2. Use a caliper to measure the diameter of the coil
3. Secure the coil vertically above the magnet using ring stand
4. Connect Linear Actuator to Arduino, L298N motor controller, and power source. Mount linear actuator on the table facing upward, towards the coil
5. Attach the Neodymium magnet to the linear actuator with 3D-printed adapter
6. Run the linear actuator with a constant pulse-width modulation signal.
7. Attach multimeter probes to opposite ends of the coil, set it to measure DC potential difference
8. Set up cell phone camera to record the multimeter screen and the coil
9. Run linear actuator with constant PWM signal, stop linear actuator and cell phone video once the magnet starts exiting coil
10. Reset linear actuator so the magnet is below the coil
11. Review the video to find the peak potential difference displayed and record it.
12. Repeat steps 6-11 for five total trials.
13. Repeat steps 2-12 for the other four coils

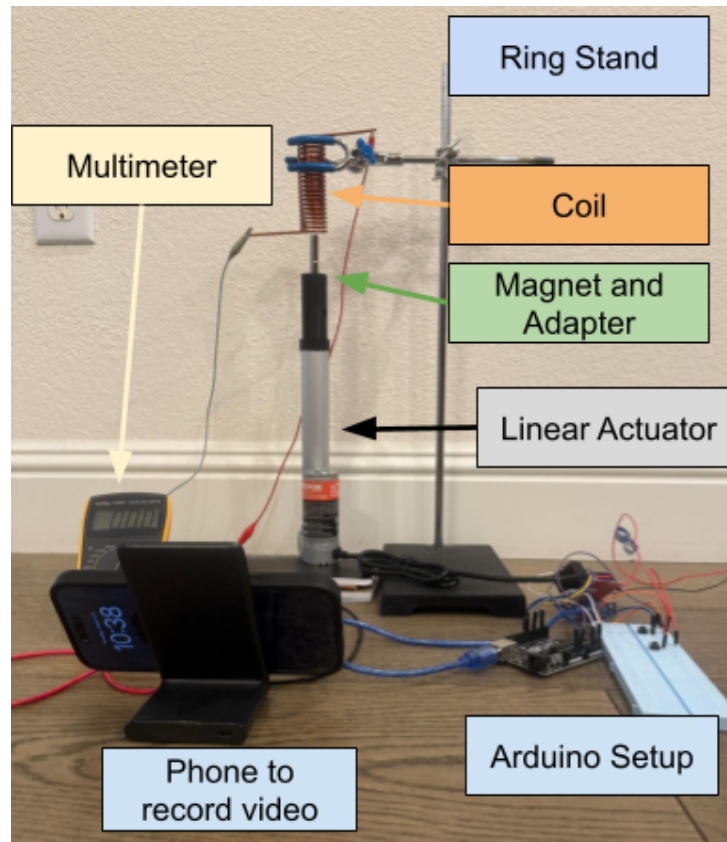


Figure 1: annotated photograph of the setup of the procedure taken by the student. Not pictured: DC power supply

Limitations of the Methodology

Uniformity of Magnetic Field

The magnetic field produced by the neodymium magnet might not be perfectly uniform along the length of the coil, even if passed through the coil at a constant speed. As mentioned earlier, this could be due to variations of the magnetic field strength with distance. This could cause a drop off in measured EMF for larger coils.

Coil Geometry Variations

Although all of the coils were manufactured to be identical in material and number of turns, small differences in coil shape or spacing/pitch between turns could affect the resistance and the magnetic flux linkage, which could cause minor inconsistencies in the induced EMF that are not attributed solely to changes in coil radius.

Actuator Speed and Magnetic Field

The linear actuator must move at a constant and identical speed across all trials conducted across all coils to ensure a constant $\frac{\Delta B}{\Delta t}$. However, while the PWM signal received by the linear actuator is precise, there could be slight variations in speed due to small power fluctuations, potentially introducing variability in $\frac{\Delta B}{\Delta t}$, therefore changing the calculated induced EMF in each coil. Additionally, since the actuator is powered by electricity, it may generate its own magnetic field that could influence measurements. However, by measuring the magnetic field around the linear actuator using a hall sensor, this effect was found to be extremely minimal because the magnetic field produced by the actuator's DC motor is extremely weak.

Safety/Ethics/Environmental concerns:

There are not many ethical or environmental risks raised by this experiment, but the use of neodymium magnets does introduce important safety considerations. Neodymium magnets are strong and can attract metal objects or other magnets over large distances. This strong attraction can cause high-speed collisions, potentially breaking objects and sending fragments flying outwards, risking injury for those nearby. To mitigate this, I will store the magnets in separate cardboard containers and wear safety goggles to protect my eyes from small fragments. I will keep all electronic devices far away from the magnets because the powerful magnetic fields of neodymium magnets can interfere with electronic devices and potentially cause permanent damage.

Analysis

Data Collection:

The data collected in Table 1 will be used to calculate the coil radius, coil radius squared, uncertainty in coil radius squared, and uncertainty in measured induced EMF.

Coil Diameter (mm) ± 0.01 mm	Induced EMF in Coil ϵ (mV) ± 0.1 mV					Average
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	
22.89	1.1	1.6	1.0	1.7	1.1	1.3
31.75	2.3	2.2	2.4	2.6	1.9	2.3
49.53	4.8	4.9	4.2	4.9	4.9	4.7
53.34	5.3	5.0	5.5	5.4	5.3	5.3
59.69	5.8	5.1	5.6	5.8	5.5	5.6

Table 1: Data collected by recording maximum EMF as the center of the magnet was passing through the coil

Raw data collected in the experiment is shown in Table 1. Absolute uncertainty for coil diameter is 0.01 mm because the digital caliper used to measure coil diameter measures up to two decimal places. The absolute uncertainty for the potential difference across the coils comes from the digital multimeter, which can measure millivolts up to one decimal place.

Qualitative Observations About the Raw Data

- Larger variation in smaller coils
 - For 22.89 mm and 31.75 mm diameter coils, there was significantly more variation between trials than in the three larger coils
 - As the coil diameter increased, the variability across trials decreased.
- The average induced EMF generally increased as the coil diameter increased
- Outliers were present in at least one trial for each of the coil diameters

Data Processing:

The data collected in the experiment and recorded in Table 1 was processed to calculate the values needed to prove the linear relationship suggested by the equation $\epsilon = (N\pi \frac{\Delta B}{\Delta t}) \cdot r^2$ by graphing ϵ V.S. r^2 . These included calculating the coil radius squared, the uncertainty in the coil radius squared, and the uncertainty in the induced EMF.

Coil Diameter (mm) ± 0.01 mm	Coil Radius (mm) ± 0.01 mm	Coil Radius Squared (mm ²)	Coil Radius Squared Uncertainty Δr^2 (mm ²)	Induced EMF in Coil Uncertainty $\Delta \epsilon$ (mV)
22.89	11.45	130.0	0.1	0.4
31.75	15.88	252.0	0.2	0.4
49.53	24.77	613.3	0.2	0.4
53.34	26.67	711.3	0.3	0.3
59.69	29.85	890.7	0.3	0.4

Table 2: Data processed from Table 1

Processing of raw data collected and presented in Table 1 was conducted using the following formulas.

$$r = \frac{d}{2}$$

where r is the coil radius and d is its diameter

Uncertainties in the coil radius squared (Δr^2) were calculated using this equation:

$$\Delta r^2 = 2\left(\frac{\Delta r}{r}\right) \cdot r^2$$

For example, for the 22.89 mm coil, the above equation was applied to calculate Δr^2 :

$$\Delta r^2 = 2\left(\frac{0.01 \text{ mm}}{11.45 \text{ mm}}\right) \cdot (11.45 \text{ mm})^2 = 0.1 \text{ mm}^2$$

This formula was applied for each coil diameter and recorded in the Δr^2 column of the processed data table.

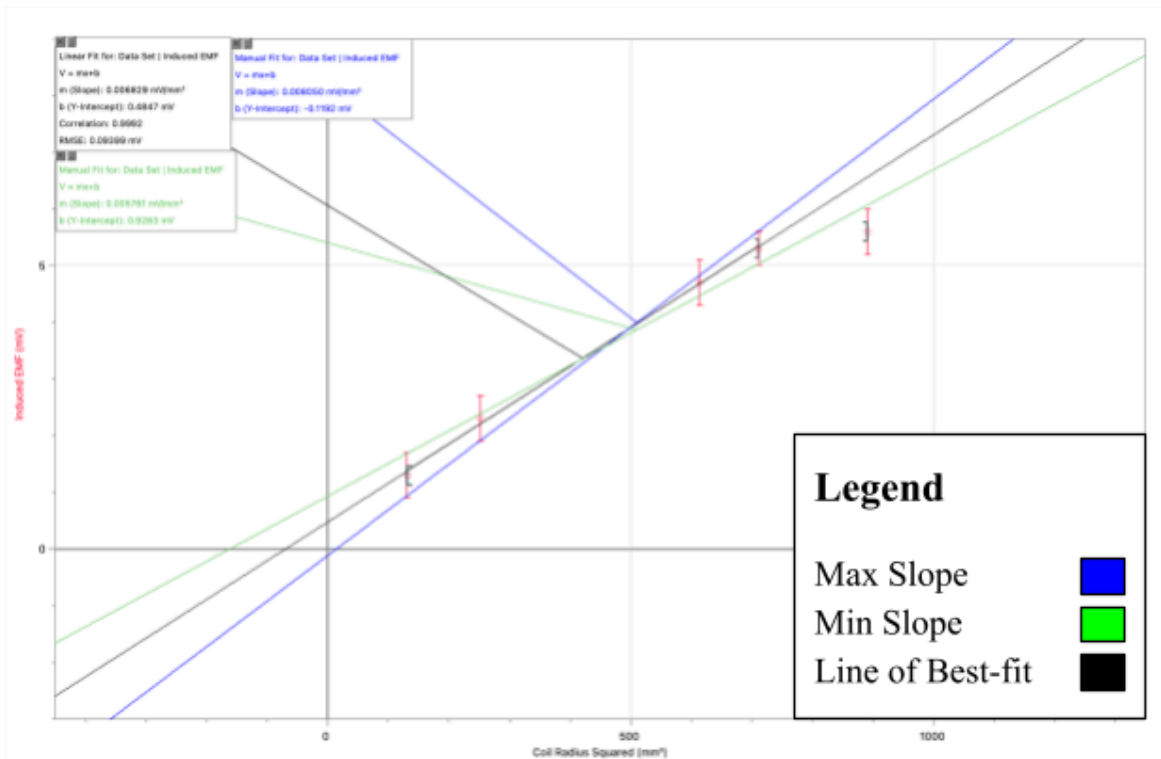
Uncertainties in maximum induced EMF $\Delta \epsilon$ were calculating using the following equation:

$$\Delta \epsilon = \frac{\epsilon_{max} - \epsilon_{min}}{2}$$

where ϵ_{max} = maximum induced EMF across the five trials for a coil of a specific diameter, ϵ_{min} = minimum induced EMF across the five trials for that coil.

Data Presentation:

The equation for the graph is $\epsilon = (N\pi \frac{\Delta B}{\Delta t}) \cdot r^2$. Induced EMF in the coil ϵ was plotted on the vertical axis and the coil radius squared r^2 as plotted on the horizontal axis.



Graph 1: Graph of Induced EMF (mV) VS Coil Radius Squared (mm²) using Vernier LoggerPro

	Maximum Slope	Minimum Slope	Line of Best-fit	Uncertainty
Slope (mV/mm²)	0.008	0.006	0.007	0.001
Y-intercept (mV)	-0.1	0.9	0.5	0.5

Table 3: Slopes and Y-intercepts of Maximum Slope, Minimum Slope, and Best-fit Lines, as well as uncertainty

A line of best-fit of Induced EMF VS Coil Radius Squared was plotted using the data points using the Vernier LoggerPro software. Maximum slope and minimum slope lines were created using manual fit and were adjusted based on the locations of the error bars and line of best-fit in order to represent the complete range of the data.

The uncertainty in the slope was calculated using the following equation, where m represents the slope, m_{max} represents the maximum slope, and m_{min} represents the minimum slope:

$$\Delta m = \frac{m_{max} - m_{min}}{2} = \frac{0.008 \text{ mV/mm}^2 - 0.006 \text{ mV/mm}^2}{2} = 0.002 \text{ mV/mm}^2$$

The uncertainty in the y-intercept was calculated using the following equation, where b represents the y-intercept, b_{max} represents the maximum y-intercept, and b_{min} represents the minimum y-intercept:

$$\Delta b = \frac{b_{max} - b_{min}}{2} = \frac{0.93 \text{ mV} - (-0.12 \text{ mV})}{2} = 0.5 \text{ mV}$$

Ultimately, the final value calculated for the slope was $0.007 \pm 0.001 \text{ mV/mm}^2$ and the y-intercept was calculated to be $0.5 \pm 0.5 \text{ mV}$. The R^2 value was 0.9992.

According to the equation, the slope of the best-fit line is equivalent to $N\pi \frac{\Delta B}{\Delta t} \cdot \frac{\Delta B}{\Delta t}$ can be calculated using the slope m .

$$\frac{\Delta B}{\Delta t} = \frac{m}{N\pi} = \frac{0.007 \pm 0.002 \text{ mV/mm}^2}{20\pi} = (1.1 \pm 0.3) \cdot 10^{-4} \text{ T/s}$$

Evaluation

Conclusion & Evaluation:

The hypothesis stated that the induced EMF would increase linearly with the square of the coil radius when a constant rate of change of magnetic field was applied by a magnet passing through the coil. The data yielded a best-fit line with a slope of $0.007 \pm 0.001 \text{ mV/mm}^2$ and a y-intercept of $0.5 \pm 0.5 \text{ mV}$. While the high R^2 value of 0.9992 indicates a strong correlation, a straight line was not the best fit for the data because it did not pass through all of the data points. The point for the largest coil (diameter 59.56 mm) is located below the best-fit line. This may be explained by the weakening of the magnetic field with distance, as magnetic field strength is inversely proportional to the cube of the distance. Parts of the largest coil may have experienced a weaker magnetic field, reducing the change in magnetic flux and, consequently, the induced EMF.

The expected slope was approximately $0.02 \pm 0.01 \text{ mV/mm}^2$, as determined prior to the experiment, which is greater than the slope of the best-fit line, $0.007 \pm 0.001 \text{ mV/mm}^2$. The actual slope does not fall within the range of uncertainty of the expected slope. Also, the calculated rate of change of magnetic field strength was $3.2 \cdot 10^{-4} \pm 10^{-4} \text{ T/s}$, but the actual rate of change of magnetic field was $(1.1 \pm 0.3) \cdot 10^{-4} \text{ T/s}$, which does not fall in the range of uncertainty of the calculated value. These suggest possible errors in the experiment.

Conceptually, a coil with zero radius has no area to capture magnetic flux, so the y-intercept should be zero. While zero falls within the range of error of the calculated y-intercept, $0.5 \pm 0.5 \text{ mV}$, the y-intercept has low precision because its error is extremely high at 100%. A possible source of error is misalignment of the magnet, especially in smaller coils, where small shifts have a greater impact on the magnetic flux linkage because the coil area is smaller. A 3D-printed aligning tool to align the magnet at and the coil vertically could mitigate this.

In addition to collecting more data, another improvement to the experiment could be expanding the range of coil sizes. Using coils larger than the current maximum coil would help verify if the deviation from a seemingly linear relationship is due to the weakening of the magnetic field with distance. Another improvement to the experiment is measuring the rate of change of the magnetic field using a Hall sensor during each trial to ensure the rate of change of magnetic field strength is constant.

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